

1. Let $A = \begin{bmatrix} 5 & -2 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 5 \\ -1 & 5 \end{bmatrix}$. For each matrix:
 - (i) Find the eigenvalues and the corresponding eigenvectors of A .
 - (ii) For each eigenvalue, write down a basis for the corresponding eigenspace.
 - (iii) Find an invertible matrix P and a matrix C of the form $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ such that the given matrix has the form PCP^{-1} .
 - (iv) Find the angle of rotation in matrix C .
2. Let A be a 2×2 matrix with a complex eigenvalue $\lambda = 3 - 4i$ and corresponding eigenvector $v = \begin{bmatrix} 1+i \\ 2 \end{bmatrix}$.
 - (i) Find the matrix A .
 - (iv) Find an invertible matrix P and a matrix C of the form $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ such that $A = PCP^{-1}$.
3. Let $A = \begin{bmatrix} 3 & 0 & -5 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$.
 - (i) Find the eigenvalues and the corresponding eigenvectors of A .
 - (ii) For each eigenvalue, write down a basis for the corresponding eigenspace.
 - (iii) Find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$.
4. Determine which pairs of vectors are orthogonal to each other.

a) $u = \begin{bmatrix} 12 \\ 3 \\ -5 \end{bmatrix}, v = \begin{bmatrix} 2 \\ -3 \\ 3 \end{bmatrix}$

b) $u = \begin{bmatrix} -3 \\ 7 \\ 4 \\ 0 \end{bmatrix}, v = \begin{bmatrix} 1 \\ -8 \\ 15 \\ -7 \end{bmatrix}$
5. Let $W = \text{Span} \left\{ \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} \right\}$.
 - a) Find the orthogonal complement of W , i.e, W^\perp .
 - b) Find a basis for W^\perp .
 - c) What are the dimensions of W and W^\perp .